

# DISPERSION CHARACTERISTICS OF ELEVATED SHIELDED STRIPLINE

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## ABSTRACT

The singular integral equation technique is used to derive the secular equation for an elevated shielded stripline. The dispersion characteristics of the quasi-TEM mode as well as higher-order modes are examined.

## Summary

The cross-section of the device analyzed in this paper is shown in Fig. 1. In this shielded stripline structure the inner conductor or strip may be elevated above the dielectric interface by a distance  $h + d$  and may be offset horizontally about the  $y$ -axis with end points at  $x = -w_2$  and  $w_1$ . Region 1 is assumed to have a relative permittivity  $\epsilon_r$  and a relative permeability  $\mu_r$  while the relative material parameters for regions 2 and 3 are both one. The width and height of the outer conductor are respectively  $2a$  and  $2b$ .

The structure shown in Fig. 1 was used as a model of the so-called "TEM cell", a device being developed by the National Bureau of Standards for EM susceptibility and emissions testing of electronic equipment [1]. In some applications [2] the TEM cell is loaded with absorbing material to reduce the  $Q$  of the higher-order modes and thus extend the cell's useful frequency range. In this paper we will examine the effect that the absorbing material (modeled by region 1) has on the dominant quasi-TEM mode as well as higher-order modes. The analysis is similar to that used by Mittra and Itoh [3] in their treatment of the shielded microstrip line.

This problem is easily formulated by expanding the fields in each region as a superposition of LSE and LSM modes. Assuming propagation according to the factor  $e^{j\omega t - \Gamma z}$  one can derive the following coupled integral equations in terms of the surface currents  $J_x$  and  $J_z$  on the inner conductor:

$$\begin{aligned} P \int_{-w_2}^{w_1} \partial_x J_x(x') G_{i1}(x, x') dx' + P \int_{-w_2}^{w_1} J_z(x') G_{i2}(x, x') dx' \\ = 0 \end{aligned} \quad (1)$$

$(-w_2 < x < w_1) \quad (i = 1, 2)$

where  $P$  denotes that the integrals are to be interpreted in the principal value sense and the kernels  $G_{ij}(i, j = 1, 2)$  are all of the form

$$G_{ij}(x, x') = \frac{A_{ij}(\Gamma) \sin \phi}{(\cos \theta - \cos \phi)} + \sum_{m=0}^{\infty} B_{ij,m}(\Gamma) \cos m\theta \sin m\phi \quad (2)$$

with

$$\theta = \frac{\pi}{2a} (x + a)$$

and

$$\phi = \frac{\pi}{2a} (x' + a)$$

Equation (2) is written in a form so that the singular parts of the kernels are easily recognized. In order to solve the integral equations given in (1) we move the parts containing the nonsingular kernels to the right-hand sides of the equations and treat them as forcing terms. Using the theory of singular integral equations [4] one can then invert the resulting integral equations and obtain

$$F_i(v) = \frac{1}{[1-v^2]^{\frac{1}{2}}} \sum_{m=0}^{\infty} C_{mi}(\Gamma) T_m(v) \quad (i = 1, 2) \quad (3)$$

where  $T_m(v)$  is a Chebyshev polynomial of the first kind,  $C_{mi}(\Gamma)$  are as yet undetermined coefficients,

$$F_1 = \partial_x J_x - \Gamma J_z$$

$$F_2 = \partial_x J_x - \Gamma \left[ 1 + \left( \frac{\Gamma}{k_0} \right)^2 \right] J_z$$

and

$$v = \frac{1}{\beta} \left[ \alpha + \sin \left( \frac{\pi x}{2a} \right) \right]$$

with

$$\alpha = \frac{1}{2} \left[ \sin \left( \frac{\pi w_2}{2a} \right) - \sin \left( \frac{\pi w_1}{2a} \right) \right]$$

and

$$\beta = \frac{1}{2} \left[ \sin \left( \frac{\pi w_2}{2a} \right) + \sin \left( \frac{\pi w_1}{2a} \right) \right]$$

One can see from (3) that the solutions have the correct edge behavior, this being a well-known property of the singular integral equation technique. The constants  $C_{mi}(\Gamma)$  determine the functional behavior of the solutions away from the edges and their determination will also lead to the secular equation from which the propagation constant  $\Gamma$  can be determined.

A matrix equation for the constants  $C_{mi}(\Gamma)$  can be found in a manner analogous to that used in [5] whose size depends upon the number of terms that are

kept in the expansions for the nonsingular kernels given in (2). Since the coefficients  $B_{ij,m}(\Gamma)$  are rapidly convergent for an elevated and/or a narrow stripline, accurate results can be obtained using only small order matrices. In the simplest case we keep only one term and obtain a first-order approximation for the propagation constant valid for small strip widths which is given simply as

$$\Gamma = j k_0 \quad (4)$$

If the stripline is located right on the dielectric interface then the coefficients  $A_{ij}(\Gamma)$  in (2) are slightly altered and instead of (4) one obtains

$$\Gamma = j k_0 \left[ \frac{1 + \epsilon_r}{1 + \mu_r^{-1}} \right]^{1/2}$$

which is identical to the result obtained by Coleman [6]. Additional results have been obtained for larger matrices which enable one to calculate the propagation constant of the quasi-TEM mode more accurately as well as to determine its dependence on strip width. In addition the propagation constants of higher-order modes can be obtained.

#### References

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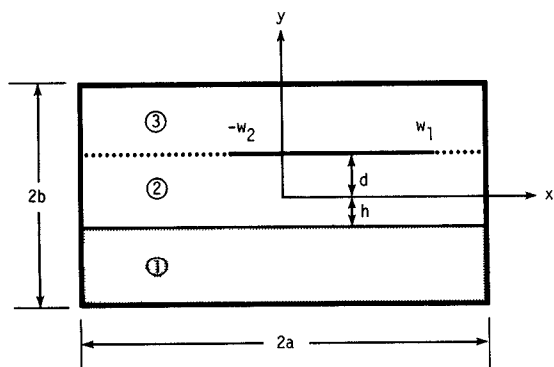


Fig. 1 Cross-section of an Elevated Shielded Stripline